Feedback Sensitivity of Distributed-Feedback Laser Diodes in Terms of Longitudinal Field Integrals

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Abstract—Using an alternative rate-equation description, analytical expressions are derived for the change of threshold gain, oscillation frequency and linewidth due to weak external optical feedback in distributed feedback laser diodes.

Index Terms—External feedback, optical communication, semiconductor lasers.

I. INTRODUCTION

THE INFLUENCE of external optical feedback on the behavior of laser diodes has been extensively studied since 1980, when Lang and Kobayashi [1] published the first experimental and theoretical analysis. The Lang–Kobayashi (LK) rate equations have, e.g., been used to derive expressions for the change of the threshold gain ΔG_{th}, of the oscillation frequency $\Delta\omega$, and of the laser linewidth $\Delta\nu$ due to weak external feedback

$$\Delta G_{\mathrm{th}} = 2\,|C_{L,R}|\,|r_{\mathrm{ext}}|\,\cos\left(\omega\tau + \arg[C_{L,R}]\right)$$

$$\Delta\omega\tau = \omega\tau - \omega_0\tau = -\,|C_{L,R}|\,|r_{\mathrm{ext}}|\,\tau\sqrt{1+\alpha^2}$$
$$\cdot\,\sin\left(\omega\tau + \arg[C_{L,R}] + tg^{-1}\alpha\right) \qquad (1)$$

$$\Delta\nu = \Delta\nu_0\left[1 + |C_{L,R}|\,|r_{\mathrm{ext}}|\,\tau\sqrt{1+\alpha^2}\right.$$
$$\left.\cdot\,\sin\left(\omega\tau + \arg[C_{L,R}] + tg^{-1}\alpha\right)\right]^{-2}$$

where τ is the round-trip time in the external cavity, α is the linewidth-enhancement factor, and $C_{L,R}$ is the feedback sensitivity for the facet on the left-hand and right-hand sides, respectively. r_{ext} is the field reflection of the external reflector.

However, the LK equations are strictly valid only for Fabry–Perot laser diodes, in which case analytical (real) expressions are readily written down for $C_{L,R}$. In general, the relations (1) are assumed to be valid also for DFB lasers, whereby the feedback sensitivities $C_{L,R}$ are now defined by these relations [2]. $C_{L,R}$ are then usually calculated numerically by deriving ΔG_{th} and $\Delta\omega$ from numerical laser simulations.

A more general and theoretical approach has been used by Nilsson and Buus [3], who showed that for all laser diodes

$$|C_L| + |C_R| = \frac{2}{\tau_{\mathrm{int}}}\,\alpha_{\mathrm{end}}L\sqrt{K_z}$$
$$\frac{|C_L|}{|C_R|} = \frac{\eta_L}{\eta_R} \qquad (2)$$

Laser | External cavity
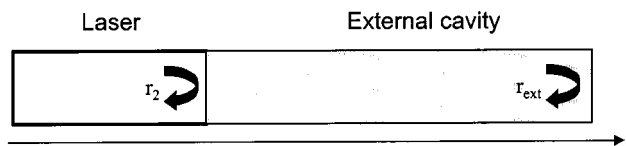
Fig. 1. Schematic structure of a laser diode with an external cavity ($\tau = L_{\mathrm{ext}}/v_g$).

where α_{end} is the mirror loss, L is the diode length, and K_z is the longitudinal Petermann factor [4]. $\eta_{L,R}$ are the extraction efficiencies for the left- and right-hand-side facets, and τ_{int} is the round-trip time in the laser cavity. Nilsson and Buus proved the relations (2) only for the influence of the feedback on the laser linewidth. Moreover, in the meantime, it has been shown that the linewidth depends on an effective linewidth enhancement factor a_{eff} and not on α. To date, it has not been clarified whether one also has to use a_{eff} to derive the influence of feedback on the change of the oscillation frequency or whether the values for $C_{L,R}$ given in (2) can be used in all of the relations (1).

In this paper, we derive exact expressions for ΔG_{th}, $\Delta\omega$, and $\Delta\nu$ based on an alternative rate-equation description for distributed feedback (DFB) lasers. The rate-equation description was published several years ago and has already been shown to lead to exact analytical expressions for the chirp and the harmonic distortion in DFB lasers [5]. Recently, we also used this rate-equation description to derive the influence of amplified spontaneous emission (ASE) on linewidth and relative intensity noise (RIN) of a laser diode integrated with a semiconductor optical amplifier (SOA) [6].

In Section II, we will derive the general, complex rate equation for a laser diode with weak external feedback. We take the facet facing the external feedback as the right-hand-side facet (at $z = L$). We then use this rate-equation description to derive exact expressions for ΔG_{th}, $\Delta\omega$, and $\Delta\nu$, and show that for a DFB laser, they can be expressed in terms of K_z and the longitudinal confinement factor.

II. RATE EQUATIONS FOR A LASER WITH EXTERNAL OPTICAL FEEDBACK

The schematic structure of the laser diode with external feedback is shown in Fig. 1. To derive the alternative rate equations in the case of weak optical feedback, we use the approach outlined in [5]. The complex amplitudes $R^+(z,t)$ and $R^-(z,t)$ of

the forward- and backward-propagating fields and the complex Bragg deviation $\Delta\beta(z,t)$ are expanded as

$$R^{\pm}(z,t) = R_0^{\pm}(z) + \delta R^{\pm}(z,t)$$
$$\Delta\beta(z,t) = \Delta\beta_0 + \delta(\Delta\beta(z,t)) \tag{3}$$

where R_0 and $\Delta\beta_0$ are the field amplitudes and Bragg deviation under static operation and in the absence of external feedback. The new rate equation can then be obtained after manipulation of the coupled wave equations (cf. [5]). Due to the external optical feedback (field reflection r_{ext}), one has to take into account modified boundary conditions at the right-hand-side facet ($z = L$) as follows:

$$\delta R^-(L,t) = r_2 e^{-2j\beta_0 L} \delta R^+(L,t)$$
$$+ t_2^2 r_{\text{ext}} R^+(L,t) e^{-2j\beta_0 L} e^{-j\omega\tau} \tag{4}$$

where β_0 is the Bragg wavelength and r_2 and t_2 are the field reflection and transmission, respectively, for the right-hand-side facet. Assuming that the spatial profiles of δR^+ and δR^- are identical to those of the static fields, we can further write

$$\delta R^{\pm}(z,t) = R_0^{\pm}(z)\left\{\sqrt{1+f(t)} - 1\right\}$$

and

$$\delta S(z,t) = S_0(z)f(t). \tag{5}$$

Here, $S(z)$ is the photon density in the laser cavity. Combination of the time-dependent coupled-wave equations for δR^+ and δR^- with the static coupled-wave equations and integration from $z = 0$ (left-hand-side laser facet) to $z = L_d$ (right-hand-side laser facet) yields the following equation:

$$j\frac{\partial\sqrt{1+f}}{\partial t} + v_g F_c - j\frac{v_g\left[R_0^+(L)\right]^2}{2\int_0^L dz R_0^+ R_0^-}$$
$$\cdot t_2^2 r_{\text{ext}}\sqrt{1+f}\, e^{-j\omega\tau} e^{-2j\beta_0 L}$$
$$= \sqrt{1+f}\left\{\Delta\omega + j\frac{1+j\alpha}{2} G'\left[\delta N_0 + \delta N_1(\chi - 1)\right]\right\}. \tag{6}$$

G' is the differential gain and $v_g F_c$ is a Langevin force representing spontaneous emission.

For the sake of simplicity, we have neglected the nonlinear gain, and the carrier density has been expanded as $N(z,t) = N_0(t) + N_1(t)[(S_0(z) - S_{\text{av}})/S_{\text{av}}]$, where S_{av} is the spatial average of S_0. The second-order moment of $v_g F_c$ is given by

$$\langle v_g F_c(t) F_c^*(t') v_g \rangle = G n_{\text{sp}} \frac{K_z}{S_{\text{av}} V_{\text{act}}} \delta(t - t') \tag{7}$$

where V_{act} is the active layer volume, G is the modal gain (per s), and n_{sp} is the inversion factor of the laser. χ is a complex factor that describes the influence of spatial hole burning and is a function of the field distributions in the laser cavity only [5]

$$\chi = \chi' + j\chi'' = \int_0^L dz \frac{S_0(z)}{S_{\text{av}}} \Gamma_z(z)$$

with

$$\Gamma_z(z) = \frac{R_0^+(z) R_0^-(z)}{\int_0^L R_0^+(z) R_0^-(z)\, dz} \tag{8}$$

where $\Gamma_z(z)$ is the longitudinal confinement factor.

The rate equation (6) has to be combined with the carrier density rate equations for the derivation of laser characteristics. We will not provide the derivation of the rate equations for N_0 and N_1 here and refer to [5].

III. SOLUTIONS OF RATE EQUATION (5)

To simplify the derivation further, we further assume that f is small (weak optical feedback, weak noise source) and linearize (6). We also consider only the low-frequency dynamics and take the real and imaginary part of (6) as follows:

$$\Delta\omega = \frac{\alpha}{2}G\left[\delta N_0 + \delta N_1(\chi' - 1)\right] + \frac{G'}{2}\delta N_1\chi''$$
$$+ \text{Re}\{v_g F_c\} + \frac{v_g\left|t_2^2 r_{\text{ext}}\left[R_0^+(L)\right]^2\right|}{\left|2\int_0^L dz R_0^+ R_0^-\right|}\sin(\omega\tau + \phi_c)$$

$$\frac{G'}{2}\left[\delta N_0 + \delta N_1(\chi' - 1 - \alpha\chi'')\right]$$
$$= \text{Im}\{v_g F_c\} - \frac{v_g\left|t_2^2 r_{\text{ext}}\left[R_0^+(L)\right]^2\right|}{\left|2\int_0^L dz R_0^+ R_0^-\right|}\cos(\omega\tau + \phi_c)$$

$$\phi_c = \arg\left[\frac{t_2^2 r_{\text{ext}}\left[R_0^+(L)\right]^2}{2\int_0^L dz R_0^+ R_0^-}\right] - 2\beta_0 L, \quad \frac{|t_2^2|\,|R_0^+(L)|^2}{\left|2\int_0^L dz R_0^+ R_0^-\right|}$$

$$= \frac{|t_2^2|\,|R_0^+(L)|^2}{\int_0^L dz\left[|R_0^+|^2 + |R_0^-|^2\right]}\frac{\int_0^L dz\left[|R_0^+|^2 + |R_0^-|^2\right]}{\left|2\int_0^L dz R_0^+ R_0^-\right|}$$

$$= \frac{P_{\text{out},r}}{P_{\text{av}}L}\sqrt{K_z}. \tag{9}$$

It can be shown [5] that, in the low-frequency regime, $\delta N_0 = \delta N_1$ such that the change in the (longitudinally averaged) threshold gain $\Delta G_{\text{th}} = G'\delta N_0$ is found to be

$$\Delta G_{\text{th}} = -\frac{2v_g|r_{\text{ext}}|}{(\chi' - \alpha\chi'')}\frac{P_{\text{out},r}}{P_{\text{av}}L}\sqrt{K_z}\cos(\omega\tau + \phi_c) \tag{10}$$

while substitution of the values of $\delta N_0 = \delta N_1$ in the equation for $\Delta\omega$ gives

$$\Delta\omega = \frac{v_g\left|t_2^2 r_{\text{ext}}\left[R_0^+(L)\right]^2\right|}{\left|2\int_0^L dz R_0^+ R_0^-\right|}$$
$$\cdot\left\{\sin(\omega\tau + \phi_c) - \frac{\alpha\chi' + \chi''}{\chi' - \alpha\chi''}\cos(\omega\tau + \phi_c)\right\}$$
$$= v_g|r_{\text{eff}}|\frac{P_{\text{out},r}}{P_{\text{av}}L}\sqrt{K_z}\sqrt{1 + \alpha_{\text{eff}}^2}$$
$$\cdot\sin\left(\omega\tau + \phi_c + tg^{-1}(\alpha_{\text{eff}})\right)$$
$$= \frac{X}{\tau}\sin\left(\omega\tau + \phi_c + tg^{-1}(\alpha_{\text{eff}})\right). \tag{11}$$

The last equality defines X. Finally, for the linewidth one finds after some calculations

$$\Delta\nu = \frac{R_{\text{sp}}K_z\left(1 + \alpha_{\text{eff}}^2\right)}{4\pi S_{\text{av}}V_{\text{act}}}$$
$$\cdot\frac{1}{\left[1 + X\sin\left(\omega\tau + \phi_c + tg^{-1}(\alpha_{\text{eff}})\right)\right]^2}. \tag{12}$$

From (10)–(12), one can conclude that similar, but nevertheless slightly different equations such as (1), are still valid for DFB lasers. That is

$$\Delta G_{\text{th}} = 2 \frac{|C_{L,R}|}{\chi' - \alpha \chi''} \cos(\omega\tau + \phi_c)$$

$$\Delta\omega\tau = \omega\tau - \omega_0\tau = -|C_{L,R}| \tau \sqrt{1 + \alpha_{\text{eff}}^2} \\ \cdot \sin\left(\omega\tau + \phi_c + tg^{-1}(\alpha_{\text{eff}})\right)$$

$$\Delta\nu = \Delta\nu_0 \left[1 + |C_{L,R}| \tau \sqrt{1 + \alpha_{\text{eff}}^2} \\ \cdot \sin\left(\omega\tau + \phi_c + tg^{-1}(\alpha_{\text{eff}})\right)\right]^{-2}$$

with

$$|C_{L,R}| = v_g |r_{\text{ext}}| \frac{P_{\text{out},l,r}}{P_{\text{av}}L} \sqrt{K_z}$$

and

$$|C_L| + |C_R| = \frac{2}{\tau_{\text{int}}} \alpha_{\text{end}} L \sqrt{K_z}. \tag{13}$$

That is, we have proven the relations (2), derived earlier by Nilsson and Buus, in another (and more exact) manner. Moreover, we have shown that one has to use the effective linewidth-enhancement factor α_{eff} [7] in the formulas for the linewidth and emission frequency change. Finally, the exact expression for the change of the threshold gain appears to be slightly different than what has so far been used.

Calculations of the effective linewidth enhancement factor, the longitudinal Petermann factor and of χ for different types of DFB laser diodes have been published before [4]–[7]. All of these parameters can be calculated with simple DFB laser models. It can easily be seen that structures which minimize the linewidth (through the effective linewidth-enhancement factor) will also minimize the sensitivity to external optical feedback.

IV. CONCLUSION

We have given an exact derivation for the changes in threshold gain, emission wavelength, and linewidth of a DFB laser due to the presence of weak external optical feedback. It has been shown that the feedback sensitivity of DFB lasers can be expressed in terms of longitudinal field averages, such as those needed to calculate the effective linewidth-enhancement factor.

Although we have only addressed the case of weak external feedback, it can be expected from the derivation that the above defined feedback sensitivity will also determine the behavior of DFB lasers under stronger feedback, and particularly feedback from reflectors at longer distances. Our derived complex field rate equation (6) can furthermore also be used to investigate the influence of feedback on the RIN.

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