# Eigenmode Modeling of Photonic Crystal Devices

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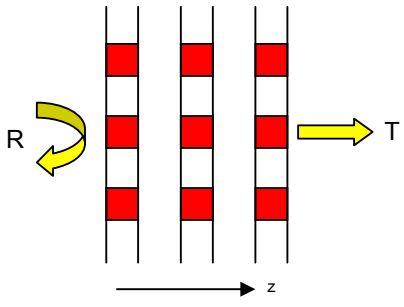
*We present a fast and efficient full vectorial modeling method for photonic crystal devices. This method is based on eigenmode expansion in z-invariant layers and is much less brute force than e.g. the standard finite-difference time-domain method. It is well suited for a large variety of structures, including infinite, semi-infinite and finite structures.*

## Introduction

Photonic crystals have received much attention lately because they are very interesting candidates for the realization of photonic integrated circuits in view of their ability to confine light, guide it around tight bends and perform a wealth of other optical functionality like filtering and add-drop multiplexing [1].
These nanophotonics devices are quite challenging to model because of their large index contrast and small dimensions. A full-vectorial model without approximations is therefore required. Candidates for this are the well-known FDTD or finite-difference time-domain method [2], but this approach is relatively brute-force and therefore quite computationally intensive. Another well-established model is plane-wave expansion [3], but it uses a super cell approach and has therefore difficulty in modeling finite devices.
The model we present here is based on vectorial eigenmode expansion [4]. We will show how this method is much less brute-force and can be used to handle infinite, finite and semi-infinite structures. It is also freely available from http://camfr.sourceforge.net.

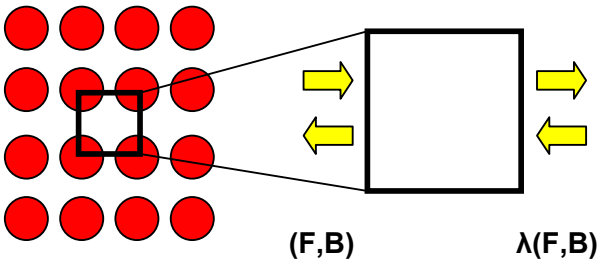
## Eigenmode expansion




Rather than using a uniform grid to spatially discretise a structure, eigenmode expansion starts by identifying layers in the structure where the refractive index profile does not change in a given direction, the so-called propagation or z-direction. In each of these layers, we expand the field in terms of the local eigenmodes of that particular layer. By applying continuity conditions and mode matching, we can deduce a scattering matrix for the structure with relates the incoming fields to the reflected and the transmitted fields. Contrary to

FDTD, calculation time is proportional to the number of layers, rather than to the total length of the structure, because propagation through a layer can be calculated analytically and is therefore independent of the layer length.

## Infinite structures




We can build on this approach to calculate the band structure of an infinite photonic crystal. We start by identifying a unit cell in the periodic structure. Bloch modes satisfy the condition that the forward and backward propagating fields at both ends 1 and 2 of the cell are proportional $(F,B)_2 = \lambda(F,B)_1$, where we can get the Bloch vector $k$ from $\lambda = \exp(-jkd)$ if $d$ is the length of the unit cell. Traditionally in literature this leads to an eigenvector formulation of the form

$$\begin{bmatrix} \mathbf{T}_{12} - \mathbf{R}_{21}\mathbf{T}_{21}^{-1}\mathbf{R}_{12} & \mathbf{R}_{21}\mathbf{T}_{21}^{-1} \\ \mathbf{T}_{21}^{-1}\mathbf{R}_{12} & \mathbf{T}_{21}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{F} \\ \mathbf{B} \end{bmatrix} = \lambda \begin{bmatrix} \mathbf{F} \\ \mathbf{B} \end{bmatrix}$$

Here **R** and **T** are the reflection and transmission matrices of the unit cell, and the subscript *ij* refers to incidence from side *i* to side *j*. However, because of the matrix inverse this formulation is numerically unstable. Therefore, we derived an alternative formulation as a generalized eigenvalue problem which does not suffer from this instability:

$$\begin{bmatrix} \mathbf{T}_{21} & \mathbf{R}_{21} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{F} \\ \lambda\mathbf{B} \end{bmatrix} = \lambda \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{R}_{12} & \mathbf{T}_{21} \end{bmatrix} \begin{bmatrix} \mathbf{F} \\ \lambda\mathbf{B} \end{bmatrix}$$

Since this method is frequency-domain method with the frequency as the independent variable, it is trivial to model dispersive media, something which is much harder using other methods.

## Semi-infinite structures

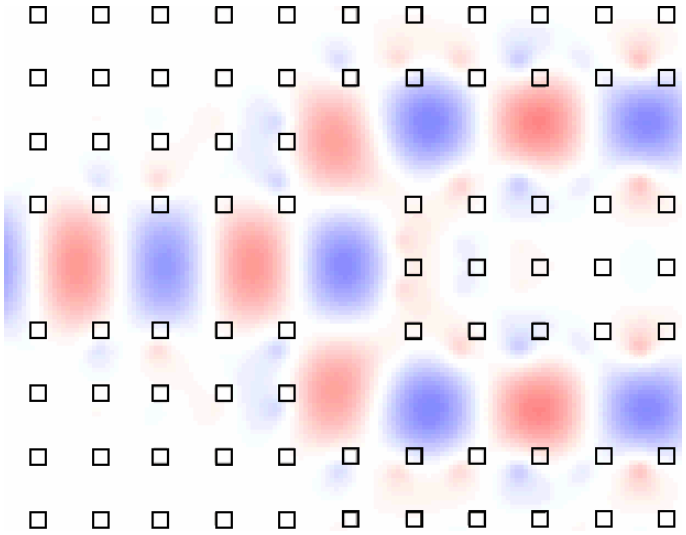
For design purposes it is often desirable to model semi-infinite structures, i.e. structures which are terminated by a semi-infinite crystal. During the design phase we can in this way eliminate parasitic reflections that would come from an imperfect termination of the crystal waveguide. To calculate the reflection matrix of a semi-infinite periodic structure, we first calculate all the Bloch modes of that structure

and retain only those with forward power flux. Each of these Bloch modes consists of forward and backward propagating components given by column vectors $\mathbf{F}_i$ and $\mathbf{B}_i$. By grouping these vectors into a matrix, one can prove that the reflection matrix of the semi-infinite crystal is given by

$$\mathbf{R}_{\text{inf}} = \mathbf{B} \cdot \mathbf{F}^{-1}$$

Note that this is just a generalization of the ratio of the reflected backward fields to the forward incident fields. Once we calculate this matrix, we can incorporate the semi-infinite crystal at the far end of another finite structure very easily by concatenating scattering matrices.

## Finite structures



Modeling finite structures does not differ substantially from the traditional eigenmode approach, but the method has a number of advantages that are especially interesting for the study of photonic crystal devices, like the splitter shown above:

- For periodic structures with a finite number of periods, the calculation time is not linear in the number of periods, but rather logarithmic. This is because the concatenation of scattering matrices behaves not unlike a multiplication, where we can e.g. speed up the calculation of $a^4$ by calculating $\left(a^2\right)^2$ rather than $a.a.a.a$.
- Because we can determine from an auxiliary calculation what the Bloch mode in the input waveguide of the splitter looks like, we can use this field profile to immediately excite the splitter. In this way, we can get an equilibrium field

distribution much quicker, which leads to a much smaller computational domain.

- FDTD first calculates the fields in the entire structure and then uses this information to deduce e.g. power transmission. In many cases however, one is only interested in the power transmission itself, in which case the calculation of the field profiles is overkill. In eigenmode expansion however, the transmission is calculated first, and the (more time consuming) calculation of the field profiles is optional.

## Conclusion

We presented a model based on vectorial eigenmode expansion which is well suited for the design of photonic crystal devices, including finite, infinite and semi-infinite structures. The modeling software is freely available from http://camfr.sourceforge.net.

## Acknowledgments

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## References

[1] J. Joannopoulos, R. Meade and J. Winn, Photonic Crystals: Molding the Flow of Light, Princeton University Press, 1995.

[2] A. Tavlove, Computational Electrodynamics - The Finite Difference Time-Domain Method, Artech House, Norwood MA, 1995.

[3] S. G. Johnson and J. D. Joannopoulos, "Block-iterative frequency-domain methods for Maxwell's equations in a planewave basis", Optics Express, vol. 8, no. 3, 173-190, 2001.

[4] P. Bienstman and R. Baets, "Optical modelling of photonic crystals and VCSELs using eigenmode expansion and perfectly matched layers", Optics and Quantum Electronics, vol. 33, no. 4/5, pp. 327-341, 2001.