

presented in section 3 and 4, is extremely sensitive to process variations. However, for the simulations presented in section 4, either currents, or detunings were swept, while keeping all other parameters the same. It is possible to compensate for a change in wavelength by changing other parameters. As an example, we illustrate how a -20 ns^{-1} detuning mismatch of the second disk with respect to the first disk (i.e., $\Delta\lambda_2 = 51 \text{ pm}$ instead of $\Delta\lambda_1 = 25.5 \text{ pm}$) can be compensated. Figure 6(a) shows how, without any compensation for this mismatch, both lasers exhibit complex self-pulsating behavior instead of the desired unidirectional pulse transfer. The reason for this complex behavior is that, for this wavelength detuning, the threshold value to obtain locking, which is also related to the SNIC bifurcation used for the excitation process, lies far above the current locking amplitude for the second laser. To see the same kind of behavior, the microdisk can be either locked using a higher amplitude, or the current can be lowered such that the bifurcation value is artificially decreased. In Fig. 6(b), the desired behavior (transfer of excitation from disk 1 to disk 2) is restored by using a combination of both strategies. The current through the second disk is decreased to 1.7 mA, and the locking power is tripled.

Using current and locking power to correct for deviations in lasing frequency, the fabrication tolerances can be relaxed with roughly a factor of 10 (a few tens of pm). This is still insufficient in view of what current fabrication techniques can achieve. Other strategies, such as deliberately changing the lasing wavelength using thermo-optic effects, can be considered. In [20], it is shown that with appropriately designed heaters, the lasing wavelength of microdisk lasers can be changed over a range of about 2 nm.

6. Conclusion

In this paper, cascading of the neuron topology introduced in our previous paper [14] was demonstrated. The choice of the phase delay in the waveguide interconnection crucially determines its excitatory (inhibitory) character by forcing destructive (constructive) interference of the output pulse of the sending disk with the locking signal of the receiving disk. When simulating a perfectly symmetrical pair of coupled disks, with a phase that corresponds to an excitatory connection, alternate oscillations going back and forth between the neurons appear. To avoid this oscillating behavior, the symmetry of the system can be broken, either by naively increasing the excitation threshold of one of the disks, or by inducing a phase difference between the locking signals of both lasers. For both methods, the lasers can be brought in a regime where only unidirectional transfer of excitation occurs. The latter behavior roughly gets lost for current variations of 0.1 mA, and frequency variations on the order of 1 ns^{-1} ($\approx 1.27 \text{ pm}$). One can however compensate for detuning variations by changing other, more controllable, parameters, such as the locking amplitude or the current, making the transfer of excitation robust to variations in lasing wavelength of several tens of pm. However, using the state-of-the-art production techniques for microdisk lasers, the standard deviation of the lasing wavelength is still about one order of magnitude larger. Additional compensation techniques, such as wavelength tuning by heating, will have to be considered.

Appendices

A. Rate equation model of two coupled microdisk lasers

A single microdisk laser can be described in the slowly varying amplitude approach using the coupled rate equations mentioned in [14], representing the evolution of the complex mode amplitudes, E^+ and E^- ($|E_{\pm}|^2$ is the number of photons in the mode, while the optical field oscillates with an additional $e^{-j\omega_m t}$ -dependency), and the number of free carriers, N , in the cavity. Caphe, the circuit simulator we use in this paper, converts the equations that describe the coupling of the optical modes to the bus waveguide into the formalism described in [17],

which, for the topology proposed in Fig. 1(a), results in the following set of coupled differential equations:

$$\frac{dE_1^+}{dt} = \frac{1}{2}(1-j\alpha)\left(G_1^+ - \frac{1}{\tau_p}\right)E_1^+ + j\Delta\omega_1 E_1^+ + CE_1^- - j\frac{\kappa\sqrt{\tau}}{\sqrt{\hbar\omega_0}}\left[-j\kappa\sqrt{\hbar\omega_0}\tau\frac{1}{\sqrt{2}}\left(e^{j\Delta\phi}\frac{1}{\sqrt{2}}E_2^-\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}E_{CW,1} + \frac{1}{\sqrt{2}}E_{tr}\right)\right], \quad (1)$$

$$\frac{dE_1^-}{dt} = \frac{1}{\sqrt{2}}(1-j\alpha)\left(G_1^- - \frac{1}{\tau_p}\right)E_1^- + j\Delta\omega_1 E_1^- + CE_1^+, \quad (2)$$

$$\frac{dE_2^+}{dt} = \frac{1}{2}(1-j\alpha)\left(G_2^+ - \frac{1}{\tau_p}\right)E_2^+ + j\Delta\omega_2 E_2^+ + CE_2^- - j\frac{\kappa\sqrt{\tau}}{\sqrt{\hbar\omega_0}}\left[-j\kappa\sqrt{\hbar\omega_0}\tau\frac{1}{\sqrt{2}}\left(e^{j\Delta\phi}\frac{1}{\sqrt{2}}E_1^-\right) + \frac{1}{\sqrt{2}}E_{CW,2}\right], \quad (3)$$

$$\frac{dE_2^-}{dt} = \frac{1}{\sqrt{2}}(1-j\alpha)\left(G_2^- - \frac{1}{\tau_p}\right)E_2^- + j\Delta\omega_2 E_2^- + CE_2^+, \quad (4)$$

$$\frac{dN_i}{dt} = \frac{\eta I_i}{q} - \frac{N_i}{\tau_c} - G_i^+ |E_i^+|^2 - G_i^- |E_i^-|^2, \quad (5)$$

$$G_i^\pm = \frac{\Gamma g_N (N_i - N_0)}{1 + \Gamma \varepsilon_{NL} (|E_i^\pm|^2 + 2|E_i^\mp|^2)}. \quad (6)$$

In Eqs. (1)–(4), α is the line broadening factor, τ_p the photon lifetime in the cavity, τ is the roundtrip time of the cavity, $\Delta\omega = \omega_m - \omega_i$ the detuning between the input light ω_m and the free-running cavity frequency ω_i of disk i ($i \in \{1, 2\}$, unless otherwise mentioned $\omega_1 = \omega_2 = \omega_0$), C is the complex intermodal coupling coefficient. κ is the coupling between the disks and the waveguide. $E_{CW,i}$ are the complex amplitudes of the optical inputs used for the locking of both disks, while E_{tr} is the complex amplitude of the input pulse (in both cases $|E_\alpha|^2$ is the power in the waveguide). $\Delta\phi$ is the phase difference due to the interconnecting waveguide. The factors $\frac{1}{\sqrt{2}}$ are due to the power loss in the splitters. Equation (5) describes the evolution of the number of free carriers N_i . I_i is the injected current to each disk, q the elementary charge, η a current efficiency factor, and τ_c the carrier lifetime. G_i^\pm are the gain coefficients of the modes, g_N is the differential gain, N_0 the transparency threshold of free carriers and Γ the confinement factor. The denominator in Eq. (5) includes cross- and self-gain modulation, ε_{NL} is called the nonlinear gain suppression coefficient. The complex amplitude of the output of disk i towards the connecting waveguide can be calculated using:

$$E_{out,i} = -j\kappa\sqrt{\hbar\omega_0}\tau e^{j\Delta\phi}\frac{1}{\sqrt{2}}E_i^-. \quad (7)$$

Similar to the intermodal coupling C , this results in an intercavity coupling term $K = -\frac{\kappa^2\tau}{2}e^{j\Delta\phi}$, coupling the suppressed mode of a disk to the strong mode of the other disk. Given $\frac{|K|}{|C|} = 1.80 > 1$, the coupling is clearly stronger than the weak coupling regime as used so far for two coupled excitable SRLs with asymmetric intermodal coupling [3].

B. Numerical details of the simulations

Table 1 summarizes the model parameters used in this paper, representing a typical microdisk.

Table 1. Model parameters are taken from [14].

Parameter	Symbol	Value	Unit
Resonance wavelength	$\lambda_0 = \frac{2\pi c}{\omega_0}$	1.55	μm
Line broadening factor	α	3	
Photon lifetime	τ_p	4.17	ps
Radius microdisk	R	5	μm
Cavity roundtrip time	τ	350	fs
Intermodal coupling	C	$0.449 + 2.82j$	GHz
Amplitude coupling to the waveguide	κ	171.4	GHz
Current efficiency	η	0.5	
Group velocity of the mode	v_g	$8.82 \cdot 10^7$	$\frac{\text{m}}{\text{s}}$
Carrier lifetime	τ_c	600	ps
'Effective' differential gain	Γg_N	982.3	kHz
'Effective' nonlinear gain suppression	$\Gamma \varepsilon_{NL}$	$1.96 \cdot 10^{-6}$	
Transparency carrier amount	N_0	763500	

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