Coherent Ising machines on photonic integrated circuits

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***Abstract*—Ising machines offer high computational capabilities as next-generation hardware accelerators for combinatorial optimization problems. Here, we present numerical simulations of coherent Ising machines (CIMs) in silicon photonics, having a fifth-order nonlinearity operated in the large-noise regime. They show competitive scalability compared to other state-of-the-art CIMs.**

I. Introduction

The Ising model, known for its versatility in both theoretical and practical contexts, stands as a fundamental pillar in statistical mechanics. It provides invaluable insights into the behavior of interacting particles in various physical systems. It is depicted as an undirected graph, comprising a collection of binary Ising spin nodes ($x_i = \pm 1$) interconnected by Ising coupling interactions denoted as J_{ij}. The Ising Hamiltonian is mathematically expressed as $H = -\frac{1}{2}\sum_{i\neq j} J_{ij}x_ix_j$. When solving an Ising problem, the primary objective is to determine spin configurations that minimize the Ising Hamiltonian, given a specific coupling interaction matrix. A generic Ising problem belongs to the category of NP-hard combinatorial optimization problems, indicating that it cannot be solved in polynomial time with respect to the problem size N (i.e. the number of nodes in the Ising model) and thus requires exponential computation time to solve on a classical computer. Due to the NP-hard nature, Ising problems have the potential to encode other NP-hard combinatorial optimization problems such as Maxcut problems and traveling salesman problems.

The operational principle of an Ising machine is to solve Ising problems by implementing an Ising model and minimizing its Ising Hamiltonian over time. Figure 1 provides an example of the workflow of using an Ising machine as a Maxcut solver.

In this work, we present the simulation of a novel designed Integrated Ising machine model that can be fabricated on photonic integrated circuits. The main novelty lies in the consideration of a fifth-order nonlinearity to have more hyperparameters and a large noise regime to facilitate exploration.

II. Integrated CIM model

As an important type of Ising machine, considerable attention has been paid to coherent Ising machines (CIMs) [1]–[3], which consist of a network of optical Ising nodes with bistable states of coherent light pulses (e.g., 0 and π phases, or the in-phase amplitude of the coherent light pulses). However, most of the CIMs are based on optical parametric oscillators (OPOs), requiring large experimental setups. As a compact Ising machine implementation, integrated CIMs have been proposed in recent work [4]. A schematic of an integrated CIM on a photonic integrated circuit can be seen in Figure 2.

The time differential equation for each spin amplitude in the integrated CIM system can be approximated as a fifth-order polynomial transfer function [4]:

$$\dot{x_i} = (r-1)x_i - \eta x_i^3 + \zeta x_i^5 + \beta \sum_{j=1}^{N} J_{ij}x_j, \quad i = 1, 2, \cdots, N. \tag{1}$$

Here, x_i represents the i_{th} light amplitude or analog spin amplitude with the sign indicating the binary spin direction. r is the linear gain, η and ζ are the third-order and fifth-order coefficients, respectively. r, η, and ζ are tunable hyperparameters in our fifth-order CIM model, which can be further optimized for specific problems. The fifth-order nonlinearity comes from the nonlinearity of the microrings on the photonic circuit [4] and enables supercritical and subcritical operation regimes with hysteresis in the bifurcation diagram. Hysteresis is a phenomenon in which a system experiences a time lag in response to external changes in input or operating conditions. This lag can be likened to a ”memory effect” of the system, which can potentially enhance the system’s robustness against external random noise. Therefore, we will engineer the hysteresis by tuning the Ising machine hyperparameters in a large noise regime, to exploit its potential computational power.

III. Results

To demonstrate the computational capability of the fifth-order model in terms of scalability, we have performed numerical experiments on benchmark Maxcut problems from the BiqMac library [5]. After the optimization of the Ising machine hyperparameters, the relative success rate (SR) results shown in Figure 3 illustrate that the fifth-order CIM model outperforms the state-of-the-art OPO CIM model in 23 out of 30 Maxcut instances with an average of 60% increased

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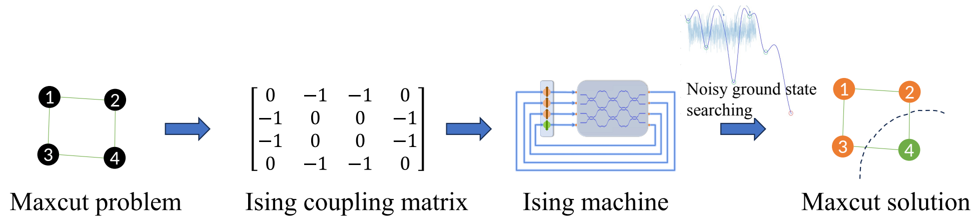

Fig. 1. Workflow of an Ising machine as a Maxcut solver: starting from a Maxcut problem defined by a certain undirected graph, the problem is then encoded into an Ising coupling matrix, which will be fed into an Ising machine with optimized hardware to this specific problem. By the noisy ground state search process, if the Maxcut problem is solvable, after an evolution time, the Maxcut solution will be finally returned as the ground-state Ising spin configuration.

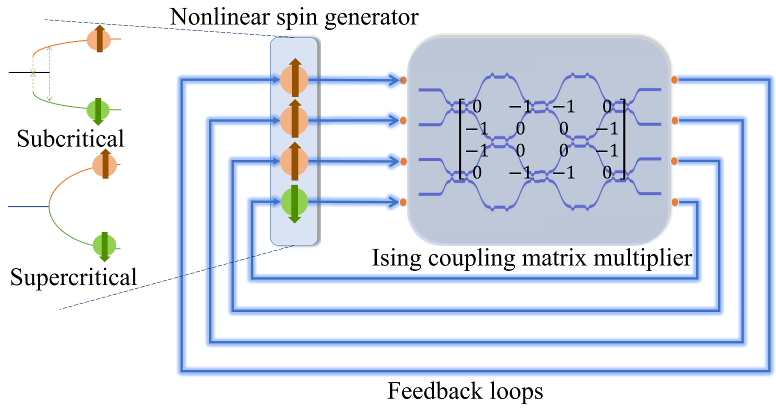

Fig. 2. Schematic of an integrated CIM on a photonics integrated circuit: three main building blocks are involved, including a nonlinear spin generator, an Ising coupling matrix multiplier, and feedback loops. The Ising machine can either be operated in subcritical pitchfork regime or supercritical pitchfork regime. The nonlinear spins and matrix multiplier can be realized by microring resonators and MZI mesh on a photonic integrated circuit.

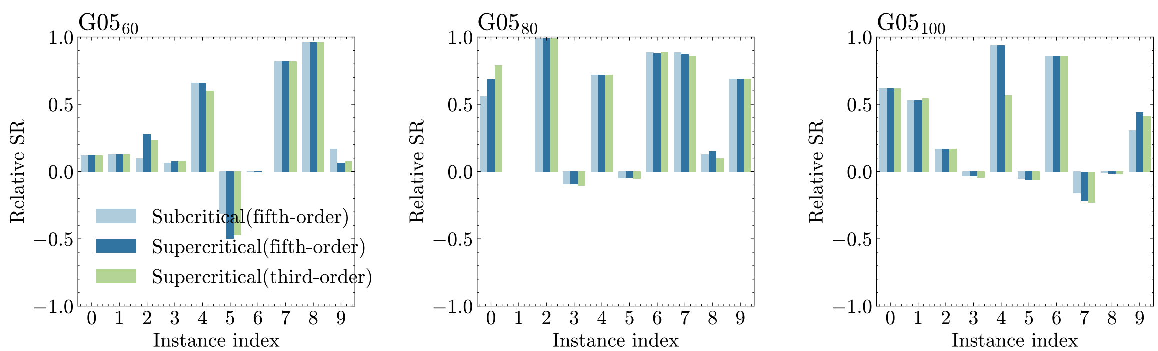

Fig. 3. Relative success rate (SR) for Biqmac Maxcut instances with $N = 60, 80, 100$ nodes and 50% edge density, compared with simulation results of the state-of-the-art CIM in [6]. The SR refers to the probability of finding the ground state during one simulation trial, which was obtained by 100 trials per instance.

SR. The advantages are more pronounced in large instances with $N = 80, 100$. This indicates that involving more tunable hyperparameters and enabling hysteresis with large noise in the optimization process enhances the computational power of the Ising machine.

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